105TH PERIPATETIC SEMINAR ON SHEAVES AND LOGIC

(SATELLITE EVENT: CATEGORICAL METHODS IN ALGEBRA)

DIPARTIMENTO DI MATEMATICA E INFORMATICA
UNIVERSITÀ DEGLI STUDI DI PALERMO
(17th) 18th AND 19th MAY 2019

Programme

Friday 17 May - Categorical Methods in Algebra - satellite event

14:30	S. Paoli	Comonad cohomology of track categories
15:30	*Coffee break*	
16:00	X. Garcia Martinez	A characterisation of Lie algebras via algebraic exponentiation
17:00	J. Vitoria	Approximations and torsion pairs in triangulated categories
18:15	*Klein Concert: Five Compositions on a Geometric Shape, by Maria Mannone*	

Saturday 18 May - PSSL 105 first day

9:00	Welcome and opening of the conference.	
9:20	J. Adamek	Algebraically Cocomplete Categories
9:55	L. Sousa	Criterions for finitarity
10:30	*Coffee break*	
11:00	D. Bourn	n-nilpotent objects and n-folded objects
11:35	T. Van der Linden	On the "Three Subobjects Lemma" and its higher-order generalisations
12:10	J. Gray	On the representability of actions of the category of internal categories of a semi-abelian category
12:45	*Lunch break*	
14:45	P. Rosolini	Relating the Effective Topos to Homotopy Type Theory
15:20	M. E. Maietti	A predicative version of the tripos-to-topos construction
15:55	M. Rogers	Monoids and their Toposes
16:30	*Coffee break*	
17:00	J. Xarez	A possible extension of the (trivial) covering morphisms for commutative semigroups via semilattices
17:35	C. Rizzo	Differential identities and almost polynomial growth
20:30	*Conference dinner*	

Sunday 19 May - PSSL 105 second day

9:00	G. Janelidze	What is the spectral category?
9:35	N. Martins-Ferriera	Internal categorical structures in cancellative conjugation semigroups and monoids
10:10	*Coffee break*	
10:40	L. Santocanale	A category of words and paths
11:15	J. Lindberg	Constructive semantics and the Joyal-Tierney representation theorem
11:50	W. Tholen	Abandoning monomorphisms: partial maps, fractions and factorization

Abstracts (satellite event)

Xabier García-Martínez, Universidade de Vigo.

A characterisation of Lie algebras via algebraic exponentiation.

From a categorical-algebraic perspective, the category of Lie algebras over a fixed ring or field has some very interesting properties such as *peri-abelianness* [3], *strong protomodularity* [2], the *Smith is Huq* condition [11], *normality of Higgins commutators* [5], and *algebraic coherence* [4]. Moreover, James Gray defined an even stronger property, which he called *locally algebraically cartesian closedness* (LACC for short) [9], which implies all the previous ones, while still having Lie algebras as an example [10].

Being aware of its important implications, we started looking for any other varieties of non-associative algebras which satisfy it, but we ended up finding something more shocking: in the setting of non-associative algebras, LACC turns out to be a distinguishing property of Lie algebras [7,8]. More explicitly, we proved the following theorem:

> Theorem. Let $\mathbb{K}$ be an infinite field. Let $\mathcal{V}$ be a variety of n-algebras over $\mathbb{K}$ which is a non-abelian locally algebraically cartesian closed category. Then $n = 2$ and
>
> (1) if $char(\mathbb{K}) \neq 2$, then $\mathcal{V}$ is the variety of Lie algebras;
>
> (2) if $char(\mathbb{K}) = 2$, then $\mathcal{V}$ is either the variety of Lie algebras or the variety of quasi-Lie algebras.

To prove this, we needed to make use of a computer, since at some point we found a rather large set of polynomial equations of degree 3 and we needed to see whether or not they have a solution in the given field. Since doing it by hand was not a viable solution, we used the theory of Gröbner bases, and our argument became the first computer assisted proof in categorical algebra.

Currently we are exploring some other strong categorical-algebraic properties and their relations with Lie algebras. In particular we are trying to understand the role of *representability of actions* [1] in the non-associative algebras setting.

This talk will be divided in three parts. In the first one, we will go through our characterisation of Lie algebras explaining the definitions and an outline of the proof. Then we will look carefully at the computational part and explore the theory of Gröbner bases theory a bit (The book [6] is a very good reference for this). Finally we will report on state of our current investigation involving representability of actions.

Joint work with Tim Van der Linden and Corentin Vienne.

References

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[7] X. García-Martínez and T. Van der Linden, A characterisation of Lie algebras amongst alternating algebras, J. Pure Appl. Algebra (2019), in press.

[8] X. García-Martínez and T. Van der Linden, A characterisation of lie algebras via algebraic exponentiation, Adv. Math. 341 (2019), 92–117.

[9] J. R. A. Gray, Algebraic exponentiation in general categories, Ph.D. thesis, University of Cape Town, 2010.

[10] J. R. A. Gray, Algebraic exponentiation for categories of Lie algebras, J. Pure Appl. Algebra 216 (2012), 1964–1967.

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Simona Paoli, University of Leicester.

Comonad cohomology of track categories.

Simplicial categories are one of the models of $(\infty, 1)$-categories. They can be studied using the Postnikov decomposition, whose sections are categories enriched in simplicial n-types and whose k-invariants are defined in terms of the (S, O)-cohomology of Dwyer, Kan and Smith. The latter is defined topologically, while the understanding of the k-invariants calls for an algebraic description. In this talk I illustrate the first step of this program, for categories enriched in groupoids, also called track categories. We define a comonad cohomology of track categories and we show that, under mild hypotheses on the track category, its comonad cohomology coincides up to a dimension shift, with its (S, O)-cohomology, therefore obtaining an algebraic formulation of the latter. This is joint work with David Blanc.

Jorge Vitoria, Università degli studi di Cagliari.

Approximations and torsion pairs in triangulated categories.

Triangulated categories are fundamental objects in homological algebra, and many of those naturally occurring in algebra are generated by their small (compact) objects. Key examples range from stable module categories over group rings to derived categories of modules over a ring or sheaves over a scheme. For such a triangulated category T, it is then natural to consider the abelian category of (contravariant) functors on the compact objects. This abelian category is a locally coherent Grothendieck category and it naturally induces a pure-exact structure on T, controlling some of its structural aspects. In this talk we will discuss the interactions between the pure-exact structure

mentioned above and the existence of approximations (precovers and preenvelopes) for subcategories of a compactly generated algebraic triangulated category. Moreover, we will focus on how to use the pure-exact structure to create torsion pairs, which are pairs of subcategories with special approximation-theoretic properties that allow for a useful decomposition of the underlying triangulated category. This is based on joint work with Rosanna Laking.

Abstracts (PSSL 105)

Jiří Adámek, Czech Technical University, Prague.

Algebraically Cocomplete Categories.

Freyd introduced in [1] algebraic completeness of a category: this means that every endofunctor has an initial algebra. For every uncountable regular cardinal k we prove that the category Set_k of sets of power at most k is algebraically complete and cocomplete, i.e., terminal coalgebras also exist for all endofunctors. Algebraic completeness holds in fact for all cardinals k, see [2]; however the category of countable sets is *not* algebraically cocomplete.

From this we derive that every endofunctor of Set with an initial algebra of an uncountable regular power has a terminal coalgebra of the same power. This is surprising since a number of 'everyday' set functors (e.g. $FX = X \times X + 1$) have countable initial algebras, whereas their terminal coalgebras are uncountable.

References.

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[2] J. Adámek, S. Milius, and J. Velebil. On coalgebra based on classes. Theoret. Comput. Sci. 316 (2004) 3?23.

Dominique Bourn, Université du Littoral, Calais.

n-nilpotent objects and n-folded objects (Joint work with Clemens Berger).

We tried to import inside Algebra the notion of degree of a functor introduced by Goodwillie for the *functor calculus* in Topological Algebra, and first focused our attention on the degree of the functor $Id : \mathbb{D} \to \mathbb{D}$ when $\mathbb{D}$ is a pointed regular Mal'tsev category.

First we observed that, when the functor $Id_{\mathbb{D}} : \mathbb{D} \to \mathbb{D}$ is of degree $\leq n$, then it necessarily takes values inside the subcategory n-$Nil\mathbb{D}$ of n-nilpotent objects in $\mathbb{D}$. Then we introduce the notion of a n-folded object and the subcategory $Fld^n\mathbb{D}$ of n-folded objects in $\mathbb{D}$, getting $Fld^n\mathbb{D} \subset n\text{-}Nil\mathbb{D}$. The functor $Id_{\mathbb{D}}$ is of degree $\leq n$ if and only if we have $\mathbb{D} = Fld^n\mathbb{D}$. A result of Lazar (1954) allows to show that, in the category Gp of groups, we get $Fld^nGp = n\text{-}NilGp$. A similar method produces the same result for Lie algebras, while the inclusion is strict in the case of Moufang loops.

When $\mathbb{D}$ is a semi-abelian category, the 2-folded objects are characterized by the vanishing of the Higgins commutator (in the sense of Mantovani-Metere), while the n-folded objects are characterized by the vanishing of the higher crossed effects (in the sense of Hartl-Loiseau-Van der Linden) which can be thought as a kind of higher Higgins commutators. So that, in the semi-abelian context, the higher Higgins commutators allow to measure exactly the distance separating an object X from its associated n-folded object.

James Gray, Stellenbosch University.

On the representability of actions of the category of internal categories of a semi-abelian category (joint work with M. Gran).

We show that the category of internal categories of an algebraically coherent [2] action representable [1] semi-abelian category [4] with normalisers [3] is action representable. This recovers the fact that categories of crossed modules of groups and of Lie algebras as well as the category of internal n-fold categories in groups (Loday's n-cat groups [5]) and in Lie algebras are action representable. As a corollary we see that for any category of interest C (in the sense of Orzech [6]) the category of internal crossed modules (in the sense of Porter [7]) in C is action representable as soon as C is action representable.

References.

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George Janelidze, University of Cape Town.

What is the spectral category?

In this joint work with M. J. Arroyo Paniagua, A. Facchini, and M. Gran, we explain that the well-known construction of the spectral category of an abelian category does not really need the ground category to be abelian. This also provides a categorical version of some previously obtained results of Arroyo Paniagua and Facchini on G-groups, for an arbitrary (fixed) group G.

Johan Lindberg, Stockholm University.

Constructive semantics and the Joyal-Tierney representation theorem.

In this talk I'll describe an ongoing project of further developing the constructive model theory for geometric and first-order logic using Heyting Algebra-valued sets. In particular, we study certain locales constructed from the syntax of the theory, some cases of which can be seen as analogues for geometric logic of certain formal topologies first investigated by T. Coquand with collaborators in [1], [2].

Starting from a geometric theory T, the locales X we construct are such that the geometric morphism into the classifying topos $\mathsf{Set}[T]$ from sheaves on X is an open surjection, hence these locales can be used for representing $\mathsf{Set}[T]$ in the style of Joyal and Tierney [3]. In fact, our analysis of when this geometric morphism is an open surjection allows us to identify and compare several possible locales that can be used to that end, including spatial ones (a la Butz-Moerdijk [4]) when T has enough models.

This is joint work with Henrik Forssell, Oslo University.

References.

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Maria Emilia Maietti, Università di Padova.

A predicative version of the tripos-to-topos construction.

In previous work with G. Rosolini and F. Pasquali we analyzed the tripos-to-topos construction and a tripos-to-quasi-topos construction by means of free constructions of suitable elementary doctrines.

In this talk we employ this analysis to produce a predicative version of the tripos-to-topos construction.

Nelson Martins-Ferreira, Polytechnic Institute of Leiria.

Internal categorical structures in cancellative conjugation semigroups and monoids.

We show that the category of cancellative conjugation semigroups [1] is weakly Mal'tsev and give a characterization of all admissible diagrams there. In the category of cancellative conjugation monoids we describe, for Schreier split epimorphisms with codomain B and kernel X, all morphisms from X to B which induce a reflexive graph, an internal category or an internal groupoid. We describe Schreier split epimorphisms in terms of external actions and consider the notions of pre-crossed semimodule, crossed semimodule and crossed module in the context of cancellative conjugation monoids. In this category we prove that a relative version of the so-called "Smith is Huq" condition for Schreier split epimorphisms holds as well as other relative conditions.

References.

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Carla Rizzo, Università di Palermo.

Differential identities and almost polynomial growth.

Let A be an associative algebra over a field F of characteristic zero. If L is a Lie subalgebra of the Lie algebra $\mathrm{Der}(A)$ of derivation of A, L acts on A and this action can be extended to the action of its universal enveloping algebra $U(L)$. With these ingredients at hand one studies the polynomials in non commuting variables $x^h = h(x)$, where $h \in U(L)$, vanishing in A, that is the differential identities of A. A very useful numerical invariant that can be attached to the T_L-ideal $\mathrm{Id}^L(A)$ of all differential identities of A is given by the sequence $c_n^L(A)$ of differential codimensions. In fact, this numerical sequence measures the rate of growth of the multilinear differential polynomial lying in $\mathrm{Id}^L(A)$. In [2] Gordienko showed that in case A is finite dimensional and L is a Lie algebra of derivations of A, $c_n^L(A)$ is exponentially bounded. Moreover, the differential codimensions of a finite dimensional algebra A are either polynomially bounded or grow exponentially (no intermediate growth is allowed).

In this context it is often convenient to use the language of varieties of algebras. If $\mathcal{V}$ is a variety of differential algebras, the growth of $\mathcal{V}$ is defined as the growth of the sequence of differential codimensions of any algebra with derivations A generating $\mathcal{V}$, i.e., $\mathcal{V} = \mathrm{var}^L(A)$ (the Lie algebra L acts on A as derivations). A variety $\mathcal{V}$ has almost polynomial growth if $\mathcal{V}$ has exponential growth but every proper subvariety has polynomial growth.

The purpose of this talk is to present some recent results (see [1, 3]) about the growth of the differential identities of two algebras: the algebra UT_2^D of 2×2 upper triangular matrices with the action of the Lie algebra of all derivations and the infinite dimensional Grassmann algebra $\widetilde{G}$ with the action of a finite dimensional Lie algebra of inner derivations. More precisely, we show that unlike the ordinary case UT_2^D and $\widetilde{G}$ do not generate varieties of almost polynomial growth.

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Morgan Rogers, Università dell'Insubria.

Monoids and their Toposes.

The most basic examples of Grothendieck toposes are presheaf toposes: categories of the form $[\mathcal{C}^{\mathrm{op}}, \mathsf{Set}]$. A category theorist trying to understand Grothendieck toposes for the first time will therefore naturally ask: what does such a category look like for the simplest choices of $\mathcal{C}$? In particular, what happens when $\mathcal{C}$ is a preorder or a monoid. Due to the origins of topos theory from categories of sheaves on topological spaces, where the corresponding sites are frames (that is, very structured preorders), one side of this question has been treated far more thoroughly in the topos-theory literature than the other. In this talk I shall exhibit some results regarding presheaf toposes on monoids, and then some results regarding topological monoids if time allows.

Pino Rosolini, Università di Genova.

Relating the Effective Topos to Homotopy Type Theory.

The presentation of the effective topos as an exact completion suggests that, in a very rough sense, the topos may be obtained as a quotient of some model of Homotopy Type Theory. We show that this is the case by proving that the effective topos is a full subcategory of the homotopy quotient of the category of Kan fibrant cubical assemblies. This is joint work with Steve Awodey and Jonas Frey.

Luigi Santocanale, LIS/AMU.

A category of words and paths.

A word on the two element alphabet $\{x, y\}$ describes a discrete path on the plane.

Such a word can be seen as an arrow $w\colon n \to m$, where n is the number of occurrences of x and m is the number of occurrences of y.

Indeed, there is a standard bijection from words $w\colon n \to m$ and join preserving functions from the chain $\{0, 1, \ldots, n\}$ to the chain $\{0, 1, \ldots, m\}$.

The category of words is therefore a quantaloid.

In this talk I'll describe how to include continuous paths in this framework, and how to characterize higher dimensional discrete and continuous paths as skew metrics/enrichments over this quantaloid.

Lurdes Sousa, Polytechnic Institute of Viseu and CMUC.

Criterions for finitarity

An endofunctor F in $\mathcal{S}et$ is finitary, that is, preserves filtered colimits, iff, given a set X, every finite subset of FX factorizes through the image by F of a finite subset of X. This condition has a formulation for general functors $F : \mathcal{A} \to \mathcal{B}$ between finitely presentable categories by replacing "finite subset" with "finitely generated subobject". We call the functors with this property finitely bounded. We investigate conditions under which the equivalence "finitary $\Leftrightarrow$ finitely bounded" holds.

A related topic is the characterization of finitely presentable algebras for a finitary monad $\mathbb{T}$. When the base category is $\mathcal{S}et$, they are the quotients of free algebras (TX, μ_X), with X finite, modulo a finitely generated congruence. We generalize this result to all finitely presentable categories with regular factorizations and all finitary monads preserving regular epimorphisms.

This is joint work with Jiří Adámek, Stefan Milius and Thorsten Wissmann [1].

References.

[1] J. Adámek, S. Milius, L. Sousa and T. Wissmann, On finitary functors and finitely presentable algebras, arXiv:1902.05788.

Walter Tholen, York University, Canada.

Abandoning monomorphisms: partial maps, fractions and factorization (joint work with S. N. Hosseini and A. R. Shir Ali Nasab).

For a composition-closed and pullback-stable class $\mathcal{S}$ of morphisms in a category $\mathcal{C}$ containing all isomorphisms, we form the category of $\mathcal{S}$-spans (s, f) in $\mathcal{C}$ with first "leg" s lying in $\mathcal{S}$ and, as its quotient category, give an alternative construction of the category $\mathcal{C}[\mathcal{S}^{-1}]$ of $\mathcal{S}$-fractions, the intermediate steps of which are of independent interest. Instead of trying to turn $\mathcal{S}$-morphisms "directly" into isomorphisms, we turn them separately into retractions and into sections, in a universal manner. The second of these two quotient processes leads to a legitimate candidate for playing the role of the $\mathcal{S}$-partial map category when $\mathcal{S}$ is not constrained to contain only monomorphisms of $\mathcal{C}$. Under mild additional hypotheses on $\mathcal{S}$, but still without the mono constraint, this $\mathcal{S}$-partial map category has a localization, which is a split restriction category (in the sense of Cockett and Lack), or even a split range category (in the sense of Cockett, Guo and Hofstra), and which is still large enough to have $\mathcal{C}[\mathcal{S}^{-1}]$ as its quotient. The construction of the range category is part of a global adjunction between relatively stable factorization systems and split range categories.

Tim Van der Linden, Université catholique de Louvain.

On the "Three Subobjects Lemma" and its higher-order generalisations.

We solve a problem mentioned in the article [1] of Berger and Bourn: we prove that in the context of an algebraically coherent [2] semi-abelian category, two natural definitions of the lower central series coincide.

In a first, "standard" approach, nilpotency is defined as in group theory via nested binary commutators of the form $[[X, X], X]$. In a second approach, higher Higgins commutators of the form $[X, X, X]$ are used to define nilpotent objects [3,4]

Our proof of this coincidence is based on a higher-order version of the *Three Subobjects Lemma* of [2], which extends the classical *Three Subgroups Lemma* from group theory to categorical algebra. It says that any n-fold Higgins commutator $[K_1, \ldots, K_n]$ of normal subobjects K_i of X may be decomposed into a join of nested binary commutators.

Work in collaboration with Cyrille Sandry Simeu.

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João Xarez, University of Aveiro.

A possible extension of the (trivial) covering morphisms for commutative semigroups via semilattices.

It is an open question if there is a monotone-light factorization $(\mathbb{E}', \mathbb{M}^*)$ for semigroups via semilattices (in the sense of [2]). It is also known that there is no monotone-light factorization for commutative semigroups via semilattices; not even a trivial one, since the vertical homomorphisms $\mathbb{E}$ strictly include the monotone homomorphisms $\mathbb{E}'$, but the light homomorphisms are exactly the trivial ones, that is, $\mathbb{M}^* = \mathbb{M}$ (as given in [5]). Trying to solve the open question, the reflections above were embedded into several reflections from the category of all categories, all having stable units (in the sense of [1]) which can be proved using the tools in [4]. Restricting to commutative semigroups, considering the category of families of commutative monoids (note that a semigroup can be considered as a monoid with only one invertible element), and provided the conjecture that certain morphisms are effective descent, then there would be enough stabilizing objects (in the sense of [2]), and so there would be a monotone-light factorization extending the class of light morphisms for commutative semigroups via semilattices.

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