# WOMEN IN MATHEMATICS 2024
# Recent developments in Calculus of Variations and PDE's

Università degli Studi di Palermo

Dipartimento di Matematica e Informatica

Aula 7, 7 febbraio 2024

PROGRAMMA:

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|---|---|---|
| 9.00 | Apertura dei lavori | |
| 9.15-10.00 | Florica C. Cîrstea | *Existence and classification of isolated singularities for nonlinear elliptic equations with mixed reaction terms* |
| 10.00-10.45 | Francesca Colasuonno | *Symmetry breaking for supercritical elliptic problems* |
| 10.45-11.30 | coffee break | |
| 11.30-12.00 | Ida de Bonis | *Comparison results for nonlocal singular problems* |
| 12.00-12.45 | Filomena Feo | *Recent developments on some nonlinear anisotropic fast diffusion equations* |

# Abstracts

♡ **Florica C. Cîrstea, Sydney University (Australia)**

***Existence and classification of isolated singularities for nonlinear elliptic equations with mixed reaction terms***

We present new classification results of the local behaviour of the positive $C^1(\Omega \setminus \{0\})$ solutions to nonlinear elliptic equations of the form

$$\Delta u + (2 - N - 2\rho)\,\frac{x \cdot \nabla u}{|x|^2} + \frac{\lambda u^\tau |\nabla u|^{1-\tau}}{|x|^{\tau+1}} = |x|^\theta\, u^q \quad \text{in } \Omega \setminus \{0\}, \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ is an open set containing zero. We assume that $\rho, \lambda, \tau, \theta$ and $q$ are real parameters such that $\tau \in [0, 1)$ and $q > 1$.

The case $\tau = 1$ in (1), corresponding to equations with a Hardy-type potential, has been intensively studied by many authors for special cases of $\lambda$, $\theta$ and $\rho$. Very recently, Cîrstea and Fărcăşeanu [JDE, 2021] have completely elucidated the behaviour of all positive solutions of (1) near zero, at infinity, as well as the structure of all positive solutions when $\Omega = \mathbb{R}^N$, assuming $\tau = 1$, $q > 1$ and every $\lambda, \theta, \rho \in \mathbb{R}$.

In this talk, we consider the situation $\tau \in [0, 1)$ and reveal new asymptotic profiles of the positive solutions of (1) near zero, depending on the various relations between the parameters. Using a dynamical systems approach, we establish the existence of positive radial solutions of (1), manifesting near zero some of these profiles. This talk is based on results with Aleksandar Miladinovic.

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♢ **Francesca Colasuonno, Alma Mater Studiorum Università di Bologna**

***Symmetry breaking for supercritical elliptic problems***

In this talk, I will present an existence result for the Dirichlet problem associated with the elliptic equation

$$-\Delta u + u = a(x)|u|^{p-2}u$$

set in an annulus or an exterior domain of $\mathbb{R}^N$, $N \geq 3$. Here $p > 2$ is allowed to be supercritical in the sense of Sobolev embeddings, and $a$ is a positive weight with additional symmetry and monotonicity properties, which are partially shared by the exhibited solution. In the special case of radial weight $a$, such an existence result ensures the multiplicity of nonradial solutions, provided that the inner radius of the domain is sufficiently large in terms of $p$.

In this setting, the major difficulty is the lack of compactness in a nonradial framework and a possibly unbounded domain. The proofs rely on variational techniques in invariant closed and convex sets.

This is joint work with Alberto Boscaggin (Università di Torino), Benedetta Noris (Politecnico di Milano), and Tobias Weth (Goethe-Universität Frankfurt).

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### ♣ Ida de Bonis, Sapienza Università di Roma

***Comparison results for nonlocal singular problems***

We provide symmetrization results in the form of mass concentration comparisons for fractional singular equations in bounded domains, coupled with homogeneous external Dirichlet conditions. We are referring to the following problem

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(2) \qquad \begin{cases} (-\Delta)^s u = \frac{f(x)}{u^\gamma} & \text{in } \Omega \\ u > 0 & \text{in } \Omega \\ u = 0 & \text{on } \mathbb{R}^N \setminus \Omega, \end{cases}
$$

where $\Omega$ is a bounded open set in $\mathbb{R}^N$ ($N \geq 2$), $\gamma > 0$ and $f$ is a positive summable function. The operator which appears in the left hand side is the fractional Laplacian operator.
Two types of comparison results are presented for the elliptic problem, depending on the summability of the right-hand side of the equation. The maximum principle arguments employed in the core of the proofs of the main results offer a nonstandard, flexible alternative to the ones described in Theorem 3.1 of [3]. Some interesting consequences are $L^p$ regularity results and nonlocal energy estimates for solutions. Moreover we will present some symmetrization results for the corresponding evolutive singular problem. The results presented are discussed in [1] and [2].

**References**

[1] B.Brandolini, I. de Bonis, V. Ferone and B. Volzone, *Comparison results for a nonlocal singular elliptic problem*, Asymptotic Analysis 135(3-4) (2023), 421-444.

[2] B.Brandolini, I. de Bonis, V. Ferone and B. Volzone, *Comparison results for the fractional heat equation with a singular lower order term*, preprint.

[3] V. Ferone, and B. Volzone, *Symmetrization for fractional elliptic problems: a direct approach*, Arch. Rational Mech. Anal. 239 (2021), 1733-1770.

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### ♠ Filomena Feo, Università degli Studi di Napoli “Parthenope”

***Recent developments on some nonlinear anisotropic fast diffusion equations***

In this talk, I will expose several recent results concerning the study of some nonlinear anisotropic evolution equations. In particular, the main model which will be discussed is an anisotropic version of the fast diffusion equation. The existence and uniqueness of a selfsimilar fundamental solution to this equation is shown. Moreover, the asymptotic behavior of finite mass solutions in terms of the self-similar solution will be sketched. The results are based on a recent joint work with J. L. Vázquez and B. Volzone.

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