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DICAM

NONLINEAR STOCHASTIC DYNAMICS OF AN OSCILLATING WATER COLUMN (U-OWC) HARVESTER: A FREQUENCY DOMAIN APPROACH

Pol D. Spanos[†], Federica M. Strati[*], Giovanni Malara[*] & Felice Arena[*]

[†] L. B. Ryon Chair in Engrg., G. R. Brown School of Engrg., Rice University, Houston, TX 77005.
e-mail: spanos@rice.edu.

[*] Natural Ocean Engineering Laboratory (NOEL), "Mediterranea" University of Reggio Calabria,
Loc. Feo di Vito, 89122 Reggio Calabria, Italy
e-mail: federica.strati@unirc.it, giovanni.malara@unirc.it, arena@unirc.it

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Parole chiave: Convertitori di energia da onde, U-OWC, Linearizzazione Statistica, Analisi nel dominio della frequenza.

Abstract. *This paper deals with the problem of examining the nonlinear dynamic of a U-Oscillating Water Column (U-OWC) Wave Energy Converter. The U-OWC dynamic response is governed by a set of non-linear differential equations. In the paper, an approximate linear solution is sought by using the technique of statistical linearization. The linearization scheme is implemented by identifying a surrogate linear system equivalent to the nonlinear one in a mean-square sense. In this context, frequency-domain analyses of the U-OWC response are readily implemented via standard linear input-output relationship. Comparisons between the nonlinear response computed via numerical simulations and by the approximate one assess the reliability of the method. The proposed approach is applied to a small-scale U-OWC model installed in the Natural Engineering Laboratory (NOEL) in Reggio Calabria, Italy.*

Sommario. *Il lavoro affronta il problema del calcolo della risposta di un assorbitore di energia del tipo U-Oscillating Water Column (U-OWC). Tale risposta è soluzione di un sistema di equazioni differenziali nonlineare. Nel lavoro di propone una soluzione approssimata tramite la tecnica di linearizzazione statistica. Lo schema è implementato identificando un sistema lineare equivalente al sistema originale attraverso una procedura di minimizzazione dell'errore medio quadratico. Il sistema risultante può essere risolto tramite le classiche relazioni di input-output. Confronti con simulazioni di tipo Monte Carlo confermano la validità della soluzione proposta. Il metodo è applicato per il caso di un modello in scala di un U-OWC installato presso il laboratorio naturale NOEL di Reggio Calabria, Italia.*

1 INTRODUCTION

In recent decades, the growing need of energy has fostered the development of novel technologies for harvesting clean energy from renewable resources. Among the other source, sea wave energy is a promising alternative which may contribute significantly to the worldwide supply of electrical power [1]. Various promising WEC (Wave Energy Converter) design concepts have already been proposed and developed up to the prototype stage[2]. The modelling of their dynamic performance involves, typically, non-linear differential or integral equations, and statistically described excitations due to the inherent uncertainty of ocean waves.

This paper analyzes the response of a U-OWC[3]; a prototype of this WEC is currently under construction in the port of Civitavecchia (Rome, Italy)[4]. Further, a small-scale U-OWC model is installed at the Natural Ocean Engineering Laboratory (NOEL) benign natural basin. This device belongs to the family of Oscillating Water Column (OWC)[5, 6]. Its geometrical configuration comprises a chamber containing air in the upper part and water in the lower part. The water is connected to the open wave field through a small vertical U-duct. In this system, the external wave field excites the water by inducing oscillations of the water column. Such oscillations compress and expand the air mass. Thus, an air flow from the pneumatic chamber to the atmosphere is produced through a tube containing a self-rectifying turbine. The critical feature of the U-OWC is the vertical duct. Indeed, such an element is utilized for tuning the eigen-period of the water column oscillations to a desired period.

From a mathematical perspective, the U-OWC dynamics is quite different from the traditional OWCs[7]. Indeed, OWC dynamics is adequately described by a linear model, while the mathematical description of the U-OWC involves a set of non-linear differential equations.

The aim of this paper is to derive an approximate solution of the equations governing the U-OWC dynamics via a linearization scheme. The technique is implemented in conjunction with both deterministic[8] and random problems[9-12]. In the paper, the technique is implemented and validated against numerical data pertaining to the NOEL small-scale model.

2 MATHEMATICAL BACKGROUND

2.1 Equations of motion

The dynamics of the U-OWC is described by a set of two coupled differential equations. The theoretical model was firstly developed by Boccotti[13], and then revised by Malara and Arena[14].

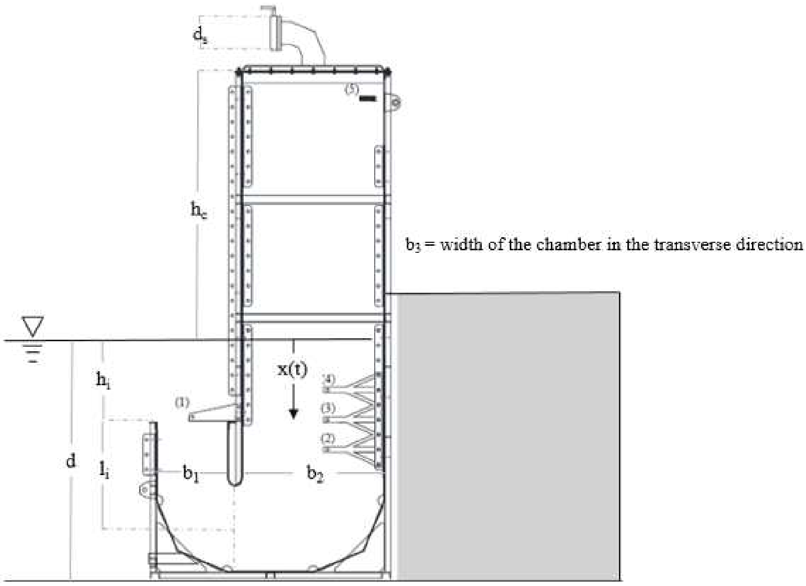

Figure 1: Cross-section of the small-scale U-OWC at the NOEL laboratory. Geometrical dimensions are: d = 1.4 m, h_i = 0.57 m, l_i = 0.8 m, b_1 =0.5 m, b_2 = 1 m, h_c =1.9 m, d_s = 0.15 and b_3=1.27 m.

Consider the schema of Fig. 1, the dimensions of the pneumatic chamber are defined by the height h_c, and the width b_2 and b_3 in the longitudinal and transverse direction, respectively. The quantities l_i and b_1 denote the length and the width of the external vertical duct, which has an opening at a water depth h_i. The hydrodynamics of the water column oscillations is described by the equation

$$\left[M(x)+\frac{1}{g}\left(\frac{b_2}{b_1}l_i+l_i+h_i\right)+C_{in}\Gamma_2\right]\ddot{x}+C(\dot{x})\,\dot{x}-x-\frac{\Delta p}{\rho g}=\eta_{ph}\ , \tag{1}$$

where x is the water level inside the pneumatic chamber measured with respect to the means water level; Δp is the pressure drop between the atmospheric pressure and pressure into the pneumatic chamber; η_{ph} is the fluctuating pressure head at the outer opening of the vertical duct; g is the gravity acceleration; and ρ is the water density. Further, the non-linear mass and damping terms are

$$M(x)=-\frac{1}{g}\left(x+C_{in}x\right), \tag{2}$$

and

$$C(\dot{x})=\frac{1}{2g}\left[-\dot{x}+C_{dg}\left|\dot{x}\right|\left(\Gamma_1-\frac{x}{R_{h,2}}\right)\right]. \tag{3}$$

with the constants Γ_1 and Γ_2 given by the equations

$$\Gamma_1=\frac{l_i}{R_{h,1}}\left(\frac{b_2}{b_1}\right)^2+\frac{l_i+h_i}{R_{h,2}}\text{, and }\quad \Gamma_2=\frac{l_i}{g}\left(\frac{b_2}{b_1}\right)^2+\frac{l_i+h_i}{g}, \tag{4}$$

and with $R_{h,1}$ and $R_{h,2}$ denoting the hydraulic radii of the U-shaped duct and inner chamber cross-section, respectively. That is,

$$R_{h,1}=\frac{b_1 b_2}{2(b_1+b_2)},\quad R_{h,2}=\frac{b_2 b_3}{2(b_2+b_3)}, \tag{5}$$

where the coefficients C_{in} and C_{dg} are experimentally calibrated to the values of 0.13 and 0.71, respectively.

Henceforth it is assumed that the excitation includes contributions due to incident, diffracted, and radiated wave fields.

The aerodynamics of the system is captured under the assumption of isentropic thermodynamic process inside the pneumatic chamber[7]. Further, the pressure drop across the orifice is linked to the mass flow rate via the non-linear equation[15]

$$\dot{m} = -C_d \; A_0 \; sign(\Delta p)\sqrt{2\rho_{air}\left|\Delta p\right|} \,, \tag{6}$$

where A_0 is the orifice area and C_d is the coefficient of discharge, which is experimentally determined equal to 0.652. The resulting governing equation is

$$\Delta\dot{p} = -\frac{c^2 C_d A_0}{b_2 b_3 \left(h_c + x\right)}\sqrt{2\rho_{air}\Delta p} - \gamma\frac{\left(\Delta p + p_{atm}\right)\dot{x}}{\left(h_c + x\right)} \,. \tag{7}$$

In a matrix notation, eq. (1) and (7) can be recast as

$$\mathbf{M_L}\,\ddot{\mathbf{q}} + \mathbf{C_L}\,\dot{\mathbf{q}} + \mathbf{K_L}\,\mathbf{q} + \boldsymbol{\beta} = \mathbf{Q} \;. \tag{8}$$

In this equation, matrices $\boldsymbol{M_L}$, $\boldsymbol{C_L}$ and $\boldsymbol{K_L}$ are the linear mass, damping and stiffness matrices respectively; the non-linear terms are captured in the vector $\boldsymbol{\beta}$; the vector $\boldsymbol{Q}$ is the excitation of the system*;* and the vector $\boldsymbol{q}$ comprises the two variable describing the displacement of the system, and its derivative. Specifically,

$$\boldsymbol{M}_L = \begin{bmatrix} \dfrac{1}{g}\left(\dfrac{b_2}{b_1} l_i + l_i + h_i\right) + C_{in}\Gamma_2 & 0 \\ 0 & 0 \end{bmatrix}, \tag{9}$$

$$\boldsymbol{C}_L = \begin{bmatrix} 0 & 0 \\ \gamma\, b_2 b_3\, p_{atm} & b_2 b_3 h_c \end{bmatrix}, \tag{10}$$

$$\boldsymbol{K}_L = \begin{bmatrix} 1 & -\dfrac{1}{\rho g} \\ 0 & 0 \end{bmatrix}, \tag{11}$$

$$\boldsymbol{\beta} = \begin{bmatrix} -\dfrac{\ddot{x}}{g}\left(x + C_{in}x\right) - \dfrac{\dot{x}^2}{2g} + C_{dg}\Gamma_1\dot{x}\left|\dot{x}\right| - \dfrac{C_{dg}}{R_{h,2}}\,x\,\dot{x}\left|\dot{x}\right| \\ b_2 b_3 x\Delta\dot{p} + +\,\gamma\, b_2 b_3\dot{x}\,\Delta p + K\,sign(\Delta p)\sqrt{\left|\Delta p\right|} \end{bmatrix}, \tag{12}$$

with the constant K given by the equation,

$$K = c^2 C_d A_0\sqrt{2\rho_{air}} \;, \tag{13}$$

while

$$\mathbf{q} = \begin{bmatrix} x \\ \Delta p \end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix} \eta_{ph} \\ 0 \end{bmatrix}. \tag{14}$$

The random excitation η_{ph} is taken as an ergodic stochastic process, characterized by a Gaussian probability density function, consistently with common sea state representation approaches[16].

2.2 Statistical linearization scheme

The authors are unaware of the exact solution to the problem (8). Thus, an approximate solution is sought via a statistical linearization scheme[12]. The scheme involves the identification of a surrogate linear system approximating the nonlinear one in a mean square sense. That is, for the nonlinear system described by the eq. (14), the equivalent linear system is

$$\left[\boldsymbol{M}_L + \boldsymbol{M}^E\right]\ddot{\boldsymbol{q}} + \left[\boldsymbol{C}_L + \boldsymbol{C}^E\right]\dot{\boldsymbol{q}} + \left[\boldsymbol{K}_L + \boldsymbol{K}^E\right]\boldsymbol{q} = \boldsymbol{Q}\,, \tag{15}$$

where $\boldsymbol{M}^E$, $\boldsymbol{C}^E$, and $\boldsymbol{K}^E$ are unknown equivalent linear matrices. These are determined by a minimization procedure on the n-vector difference $\boldsymbol{\varepsilon}$ between the original eq. (8) and the equivalent-linear system eq. (15), according to the criterion[12]

$$E\left[\varepsilon\varepsilon^T\right] = \min\,, \tag{16}$$

with E[·] being the operator of mathematical expectation. Under the approximation of Gaussian distribution of the response and of its derivative, the problem (16) leads to the equations [12]

$$m^e{}_{i,j} = E\left[\frac{\partial \beta_i}{\partial \ddot{y}_j}\right];\quad c^e{}_{i,j} = E\left[\frac{\partial \beta_i}{\partial \dot{y}_j}\right];\quad k^e{}_{i,j} = E\left[\frac{\partial \beta_i}{\partial y_j}\right], \tag{17}$$

where $k^e{}_{i,j}$, $c^e{}_{i,j}$ and $m^e{}_{i,j}$ are the coefficients of the matrices $\boldsymbol{K}^E$, $\boldsymbol{C}^E$ and $\boldsymbol{M}^E$.

3 STATISTICAL LINEARIZATION SOLUTION

The implementation of the statistical linearization technique is here illustrated. Due to the non-symmetric form of the nonlinearities[17], an offset of the steady-state response must be taken into account. Thus, the decompositions

$$x(t) = x_0 + z(t)\,, \tag{18}$$

and

$$\Delta p(t) = \Delta p_0 + h(t)\,, \tag{19}$$

where x_0 and Δp_0 are the mean of the processes x and Δp, respectively, and $z(t)$ and $h(t)$ are the zero-mean processes, are considered. Under this assumption, the original system equations must be satisfied on the average. Thus, the two averaging restoring force conditions

$$-\frac{1}{g}\left(1 + C_{in}\right)E\left[z\,\ddot{z}\right] - \frac{\sigma_{\dot{z}}^2}{2g} + x_0 - \frac{\Delta p_0}{\rho g} = 0\,, \tag{20}$$

and

$$b_2 b_3 E\left[z\,\dot{h}\right] + \gamma b_2 b_3\; E\left[\dot{z}\,h\right] + K\;E\left[sign(\Delta p)\sqrt{\left|\Delta p\right|}\right] = 0\,, \tag{21}$$

are derived, where $\sigma_{\dot{z}}$ is the standard deviation of the derivative of the water column oscillations. Eq. (20) and (21) are treated assuming a Gaussian distribution for the random process η_{ph}. Such an assumption is consistent with the typical representation of ocean waves[16]. In this context, the approximate solution of the equivalent-linear system is taken as a random Gaussian process. Eq. (17) are employed to determine the coefficients for the equivalent linear system. Accordingly, the matrices

$$\boldsymbol{M}_L{}^E = \begin{bmatrix} -\dfrac{x_0}{g}(1 + C_{in}) & 0 \\ 0 & 0 \end{bmatrix}, \tag{22}$$

$$\boldsymbol{C_L}^{\boldsymbol{E}} = \begin{bmatrix} -\dfrac{2}{g\sqrt{2\pi}} C_{dg}\left(\Gamma_1 - \dfrac{x_0}{R_{h,2}}\right)\sigma_{\dot{z}} & 0 \\ \gamma\, b_3 b_3 \Delta p_0 & b_3 b_3 x_0 \end{bmatrix}, \tag{23}$$

and

$$\boldsymbol{K_L}^{\boldsymbol{E}} = \begin{bmatrix} 0 & 0 \\ 0 & K\, sign(\Delta p_0 + h)\sqrt{|(\Delta p_0 + h)|} \end{bmatrix}. \tag{24}$$

are derived.
Clearly, an iterative procedure must be used for the computation of matrices $\boldsymbol{M^E}$, $\boldsymbol{C^E}$, and $\boldsymbol{K^E}$, as they depend on the response statistics. Thus, system response standard deviations are computed from the spectrum of the excitations S_f: using the input-output formulas

$$\sigma^2{}_z = \int_{-\infty}^{+\infty} |\alpha_{11}(\omega)|^2 S_f(\omega) d\omega; \quad \sigma^2{}_{\dot{z}} = \int_{-\infty}^{+\infty} \omega^2 |\alpha_{11}(\omega)|^2 S_f(\omega) d\omega; \quad \sigma^2{}_{\ddot{z}} = \int_{-\infty}^{+\infty} \omega^4 |\alpha_{11}(\omega)|^2 S_f(\omega) d\omega, \tag{25}$$

where α_{ij} are the components of the frequency response function of the system

$$\boldsymbol{\alpha}(\boldsymbol{\omega}) = \left[-\omega^2\left(\mathbf{M_L} + \mathbf{M_L}^{\mathbf{E}}\right) + i\omega\left(\mathbf{C_L} + \mathbf{C_L}^{\mathbf{E}}\right) + \left(\mathbf{K_L} + \mathbf{K_L}^{\mathbf{E}}\right)\right]^{-1}. \tag{26}$$

Further, the mean response values are computed via eq. (20) and (21), where

$$E[z\,\ddot{z}] = \int_{-\infty}^{\infty} \omega^2\, S_{zz} d\omega; \quad E[z\,\dot{p}] = -i\int_{-\infty}^{\infty} \omega\, S_{zp} d\omega; \quad \text{and} \quad E[\dot{z}\, p] = -i\int_{-\infty}^{\infty} \omega\, S_{ph} d\omega; \tag{27}$$

with $i = \sqrt{-1}$, S_{zz}, S_{zp} and S_{ph} being the components of the spectral density matrix of the response, and the term

$$E\left[sign(\Delta p)\sqrt{|\Delta p|}\right] \tag{28}$$

computed numerically.

Iterations are done until the equivalent matrix coefficients converge to a certain value starting from null values of the equivalent matrix coefficients.

4 NUMERICAL RESULTS

The reliability of the proposed approximate solution is assessed via preliminary comparisons with some relevant numerical data. For this purpose, the case study of the small-scale U-OWC model shown in Figure 1 is considered.
Starting from measured time histories of the excitation of the system, the response is computed numerically in the time domain. Records are sampled at a frequency of 10 *Hz* and each realization has a duration of 5 min. Given the time history of the excitation, the time-domain response is computed via the constant acceleration method[18], considering a constant time step of 0.1 s. Also, ergodic features are assumed for the U-OWC response. Quiescent initial conditions for the water column are considered in conjunction with atmospheric pressure into the air chamber. It is seen that the response statistical moments are reliably captured by the method. In this regard, note that the best agreement is observed for the second order statistics of the response. This is a desirable feature of the method, because it yields a reliable prediction of the power production when the U-OWC is implemented in conjunction with a Power – Take Off system. Pertinent results are summarized in Table 1.

H_s [m]	T_p [s]	x_0 [m]		σ_z[m]		Δp_0 [Pa]		σ_h [Pa]	
		non linear	equivalent linear	non linear	equivalent linear	non linear	equivalent linear	non linear	equivalent linear
0.50	3.30	0.001	-0.017	0.120	0.132	47	-171	633	599
0.53	3.30	0.001	-0.021	0.118	0.131	39	-216	745	756
0.50	3.31	0.004	-0.019	0.129	0.134	52	-192	725	672
0.57	3.41	-0.001	-0.016	0.114	0.116	38	-164	558	573
0.52	3.41	0.002	-0.023	0.125	0.118	47	-234	721	819
0.54	3.53	0.003	-0.021	0.120	0.130	71	-217	862	758
0.51	3.30	0.001	-0.017	0.120	0.132	47	-171	633	599
0.53	3.31	0.001	-0.021	0.118	0.131	39	-216	745	756

Table 1: Comparison between the approximate solution of the equivalent linear system and the numerical solution. The data pertain to real sea states measured at the NOEL laboratory. H_s is the significant wave height and T_p is the peak spectral period. The symbols x_0 and Δp_0 denote the mean of the water column oscillation measured with respect to the mean water level and of the air pressure fluctuations; and σ_z and σ_h are the standard deviation of the zero-mean processes z and h, respectively.

5 CONCLUDING REMARKS

An approach for determining an approximate solution to the nonlinear problem governing the response of a U-OWC device has been proposed. Case study is the small-scale U-OWC model tested at the NOEL laboratory, Reggio Calabria, Italy.

Using the statistical linearization technique, the non-linear terms have been replaced by equivalent-linear coefficients which are computed via an iterative procedure involving the computation of the response statistics. The reliability of the proposed approach has been assessed via comparisons with relevant numerical data obtained from excitations measured for NOEL model. The results show that the implementation of computationally costly numerical algorithms is circumvented efficiently by the statistical linearization technique which allows conducting readily frequency-domain analyses.

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